

Scattering lengths of strangeness $S = -2$ baryon-baryon interactions

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We reconsider a method based on dispersion theory, that allows one to extract the scattering length of any two-baryon system from corresponding final-state interactions in production reactions. The application of the method to baryon-baryon systems with strangeness $S = -2$ and $S = -3$ systems is discussed. Theoretical uncertainties due to the presence of inelastic channels with near-by thresholds are examined for the specific situation of the reaction $K^- d \rightarrow K^0 \Lambda \Lambda$ and the coupling of $\Lambda \Lambda$ to the ΞN channel. The possibility to disentangle spin-triplet and spin-singlet scattering lengths by means of various polarization measurements is demonstrated for several production reactions in $K^- d$ and γd scattering. Employing the method to available data on the $\Lambda \Lambda$ invariant mass from the reaction $^{12}\text{C}(K^-, K^+ \Lambda \Lambda X)$, a 1S_0 scattering length of $a_{\Lambda \Lambda} = -1.2 \pm 0.6$ fm is deduced.

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I. INTRODUCTION

The baryon-baryon interaction in the strangeness $S = -2$ sector and, specifically, the $\Lambda \Lambda$ system has been a topic of interest for quite some time. The fascination was generated not least by the possible existence of the so-called H -dibaryon, a deeply bound 6-quark state with $J = 0$, isospin $I = 0$, and $S = -2$, predicted by R. Jaffe in 1977 based on a bag-model calculation [1]. The binding energies of $\Lambda \Lambda$ in nuclei, deduced from sparse information on doubly strange hypernuclei [2–4] indicated a strongly attractive 1S_0 $\Lambda \Lambda$ interaction and seemed to be at least not inconsistent with the existence of such a bound state. The perspective changed drastically when in 2001 a new (and unambiguous) candidate for $_{\Lambda \Lambda}^6\text{He}$ with a much lower binding energy was identified [5], the so-called Nagara event, suggesting that the $\Lambda \Lambda$ interaction should be only moderately attractive. This conjecture concurs also with evidence provided by various searches for the H dibaryon that did not yield any support for its existence, cf. Refs. [6–8] for the latest experiments.

However, very recently the H -dibaryon was put back on the agenda. Lattice QCD calculations by the NPLQCD [9, 10] as well as by the HAL QCD [11, 12] collaborations provided evidence for a bound H -dibaryon. While the actual computations were performed for pion masses still significantly larger than the physical one, extrapolations suggest that the H -dibaryon could be still bound by around 0 – 7 MeV [13] at the physical point, but it could also move above the $\Lambda \Lambda$ threshold and dissolve into the continuum [14, 15]. (The original H -dibaryon [1] was expected to be bound by roughly 80 MeV!)

Indeed, the strength of the $\Lambda \Lambda$ interaction as well as those of other $S = -2$ baryon-baryon systems is of rather general interest, notably for a better understanding of the role played by the SU(3) flavor symmetry. Theoretical investigations of the $S = -2$ sector that have been performed within the conventional meson-exchange picture [16–21], utilizing the constituent quark model [22, 23], and also in the framework of chiral effective field theory (χ EFT) [24] all rely strongly on the SU(3) symmetry as guideline. Furthermore, the hyperon-hyperon (YY) interaction plays an important role in the understanding of the global properties of compact stars like neutron stars. Their stability and size as well as the cooling process depend sensitively on the strength of the YY interaction [25, 26].

As indicated above, practically the only experimental constraint we have so far on the $\Lambda \Lambda$ interaction comes from the analysis of double- Λ hypernuclei. In the present paper we want to call attention to the fact that there is also another and even more direct way to determine the strength of the $\Lambda \Lambda$ force but also the one in other $S = -2$ systems. It consists in studying the final-state interaction (FSI) of reactions where corresponding pairs of hyperons are produced. In fact, recently we proposed a method for extracting hadronic scattering lengths from production reactions [28–30]. The presentation of the method in those publications was done with special emphasis on its application to the hyperon-nucleon (YN) interaction. In particular, the reactions $NN \rightarrow KYN$ and $\gamma d \rightarrow KYN$ were analyzed, and possible uncertainties of the method were established. Polarization observables needed to disentangle different spin states of the final YN system were identified.

In the present paper we explore the possibility of applying the method proposed in [28–30] to the $\Lambda \Lambda$ system, but also other baryon-baryon states with $S = -2$ or even $S = -3$ are considered. Our study is motivated by the available data on the $\Lambda \Lambda$ invariant mass distribution determined in the reaction $^{12}\text{C}(K^-, K^+ \Lambda \Lambda X)$ [8, 31]. These

data are afflicted by sizeable uncertainties, but still they allow us to demonstrate the practicability of our method and to extract an actual value for the $\Lambda\Lambda$ 1S_0 scattering length. In order to stimulate future dedicated experiments we consider specifically reactions like $K^-d \rightarrow K\Lambda\Lambda$ or $K^-d \rightarrow K\Xi N$ where corresponding high-statistics measurements could be performed at J-PARC. The CLAS collaboration at Jlab has measured $\gamma p \rightarrow K^+K^+\Xi^-$ [32] and, thus, it might be feasible that they can perform also experiments for $\gamma d \rightarrow K^+K^+\Xi N$ and $\gamma d \rightarrow K^+K^0\Lambda\Lambda$ [33]. Yet another option are reactions like $pp \rightarrow K^+K^+\Lambda\Lambda$ and $pp \rightarrow K^0K^0\Sigma^+\Sigma^+$ which could be measured at the future FAIR facility, for example. Since the spin structure of these reactions differs partly from the ones considered in [28–30] the question of what polarization observables are needed to disentangle the singlet- and triplet baryon-baryon states has to be re-addressed. Note, however, that for the considered $\Lambda\Lambda$ system this issue is not relevant. Near threshold it can only be in the spin-singlet (1S_0) state. Due to the Pauli principle the other S -wave, the 3S_1 , is forbidden in this case. Thus, no polarization experiment is required and, consequently, our method could be even applied to data on $\Lambda\Lambda$ production on somewhat heavier nuclei, e.g. in $K^-^3\text{He}$ or $K^-^4\text{He}$.

Independently of that, the error estimation [28] for the method has to be re-done. Specifically, for the $\Lambda\Lambda$ system the inelastic threshold (due to the ΞN channel) lies with around 25 MeV much lower than for ΛN (where it is around 80 MeV and due to the ΣN channel). However, as we will see, the latter aspect increases the theoretical error of our method only marginally, under the discussed reasonable assumptions, and in the absence of a bound state. Taking into account the possibility of a bound state requires much more effort and is technically more complicated. In view of the current extrapolations of the lattice QCD results which rather seem to disfavor the existence of a bound state [13–15] we avoid the pertinent complications in the present study.

For completeness let us mention that FSI effects as a tool to constrain the $\Lambda\Lambda$ interaction were considered already many years ago by Afnan and collaborators [34, 35]. Their study was done under rather different presuppositions, namely for the reaction $\Xi^-d \rightarrow n\Lambda\Lambda$ and within the framework of Faddeev equations. With regard to the ΞN system there is also an entirely different possibility to determine the corresponding scattering lengths, namely via the study of Ξ^- atoms. Shifts of the energy levels due to the presence of the strong interaction would permit to deduce the scattering lengths for Ξ^-p or Ξ^-d , say, via the Deser-Trueman formula. The prospects of corresponding experiments were discussed in Ref. [36].

The paper is structured as follows. In Sect. II we review briefly our method. Section III is devoted to the $\Lambda\Lambda$ system. First we provide a new estimation for the error of the scattering length due to the extraction method, taking into account the relatively small separation of the $\Lambda\Lambda$ and ΞN thresholds. Then we apply our method to available data on the $\Lambda\Lambda$ invariant mass spectrum from a measurement of the reaction $^{12}\text{C}(K^-, K^+\Lambda\Lambda X)$. In Sect. IV we discuss several aspects of applying our method also to the ΞN and $\Sigma\Sigma$ final-state interactions and even to the strangeness $S = -3$ sector. The paper ends with a short summary. Details of the polarization observables required for separating the spin singlet and triplet states are summarized in an appendix.

II. REVIEW OF THE METHOD

The basic idea of the method is to exploit the scale separation between a short-ranged production operator and a long-ranged final-state interaction (FSI). In this case the production operator can be regarded as point-like, and the FSI can be factored out. These conditions restrict the class of reactions and kinematic regimes that one can consider. Namely, one can only apply the method to reactions with large momentum transfer q_t . Furthermore, the scattering length a in the system under consideration must have an appropriate magnitude, i.e. fulfil the condition $a \gg 1/q_t$. Sufficiently large scattering lengths are expected in the baryon-baryon sector. In particular, it is interesting to study the hyperon-nucleon and hyperon-hyperon interactions with different strangeness content of the hyperons. An elegant way to utilize the condition of scale separation is a dispersion-relation approach. Imposing unitarity and analyticity constraints on the amplitude and assuming that there are no bound states, one arrives at the following expression for the reaction amplitude A_S [28, 37, 38]

$$A_S(s, t, m^2) = \exp \left[\frac{1}{\pi} \int_{m_0^2}^{m_{max}^2} \frac{\delta_S(m'^2)}{m'^2 - m^2 - i0} dm'^2 \right] \Phi(s, t, m^2), \quad (1)$$

where m is the invariant mass of the produced baryon-baryon system with the threshold value m_0 , s is the total center-of-mass (CM) energy squared, and t represents all the remaining kinematic variables the amplitude depends upon. The function $\Phi(s, t, m^2)$ slowly varies with m^2 , which is a consequence of the assumed large momentum transfer. The cut off m_{max} has to be determined in such a way that the integral extends over the whole region where FSI effects are expected to be important. Based on scale arguments a condition for m_{max} was derived in Ref. [28] which reads, re-formulated in terms of the maximum kinetic energy in the two-baryon system, $\epsilon_{max} = m_{max} - m_0 \gtrsim \frac{1}{2a_S^2\mu}$. Here a_S is the scattering length in question and μ is the reduced mass of the baryon-baryon system. As argued

in Ref. [28], for the hyperon-nucleon interaction a typical cut off is given by the condition $\epsilon_{max} \approx 40$ MeV. The baryon-baryon scattering process has to be elastic in this region (i.e. there should be no other open channels) and it should be dominated by the s -wave amplitude parametrized by the phase shift $\delta_S(m^2)$. Note that formula (1) can only be applied to amplitudes for a specific baryon-baryon spin state S . Therefore, one has to be able to separate spin-singlet and spin-triplet states experimentally. The index “S” on the quantities above (and below) is a reminder that one has to consider the production of the baryon-baryon system in a definite spin S .

It was shown in [28] how one can invert Eq. (1) to express the scattering length via the reaction amplitude squared (or the differential cross section $\frac{d^2\sigma_S}{dm'^2dt}$)

$$a_S = \lim_{m^2 \rightarrow m_0^2} \frac{1}{2\pi} \left(\frac{m_a + m_b}{\sqrt{m_a m_b}} \right) \mathbf{P} \int_{m_0^2}^{m_{max}^2} dm'^2 \sqrt{\frac{m_{max}^2 - m^2}{m_{max}^2 - m'^2}} \times \frac{1}{\sqrt{m'^2 - m_0^2} (m'^2 - m^2)} \log \left\{ \frac{1}{p'} \left(\frac{d^2\sigma_S}{dm'^2dt} \right) \right\}, \quad (2)$$

where m_a and m_b are the masses of the two baryons, $m_0 = m_a + m_b$, p' is the CM momentum in the baryon-baryon system and \mathbf{P} indicates that the principal value of the integral has to be taken. An analogous equation can be derived for the effective range.

Possible theoretical uncertainties of the method originate from the following sources: (i) energy dependence of the production operator, (ii) influence of scattering at higher energies ($m > m_{max}$), (iii) contributions from inelastic channels (e.g. from the $\Sigma N \leftrightarrow \Lambda N$ transition) and (iv) final state interaction among other pairs of particles. For the hyperon-nucleon FSI the theoretical uncertainty in the determination of the scattering length was estimated to be 0.3 fm at most [28]. This estimate was confirmed by model calculations of production amplitudes using several different models for the hyperon-nucleon interactions with triplet and singlet scattering lengths varying from -0.7 to -2.5 fm.

The general form of Eq. (1) admits approximations under certain conditions. One of the standard approximative treatments follows from the assumption that the phase shifts are given by the first two terms in the effective range expansion,

$$p \cot(\delta(m^2)) = -\frac{1}{a} + \frac{r_e}{2} p^2, \quad (3)$$

over the whole energy range, which is usually called the effective range approximation (ERA). In this case the relevant integrals (1) can be evaluated in closed form as [39]

$$A(m^2) \propto \frac{(p^2 + \alpha^2)r_e/2}{-1/a + (r_e/2)p^2 - ip}, \quad (4)$$

where $\alpha = 1/r_e(1 + \sqrt{1 - 2r_e/a})$. Because of its simplicity Eq. (4) is often used for the treatment of the FSI. A further simplification can be made if one assumes that $a \gg r_e$, a situation that is realized in the 1S_0 partial wave of the NN system. Then the energy dependence of the quantity in Eq. (4) is given by the energy dependence of the elastic amplitude

$$A(m^2) \propto \frac{1}{-1/a + (r_e/2)p^2 - ip}, \quad (5)$$

as long as $p \ll 1/r_e$. Therefore one expects that, at least for small kinetic energies, NN elastic scattering and particle production reactions with a NN final state exhibit the same energy dependence [39–42], which indeed was experimentally confirmed for meson production [43]. The treatment of FSI effects based on Eq. (5) is often referred to as Migdal-Watson approach [40, 41], the one utilizing Eq. (4) as Jost-function approach. The reliability of such approximations as compared to the formula (2) was investigated in detail in [29]. In general the method based on Eq. (2) works systematically better than the approximations and gives scattering lengths within 0.3 fm accuracy even for rather large scattering lengths like those for NN scattering. The uncertainty in the extraction employing the other two methods is typically larger. As demonstrated in [29] these procedure lead to a systematic deviation from the true values of the scattering lengths of the order of 0.3 fm (Jost) and of 0.7 fm (Migdal-Watson).

III. THE $\Lambda\Lambda$ SCATTERING LENGTH

The $\Lambda\Lambda$ system is certainly the most promising case where one could apply our method. Its threshold is the lowest one among all $S = -2$ channels and measurements could be performed for the reaction $K^-d \rightarrow K^0\Lambda\Lambda$, for example.

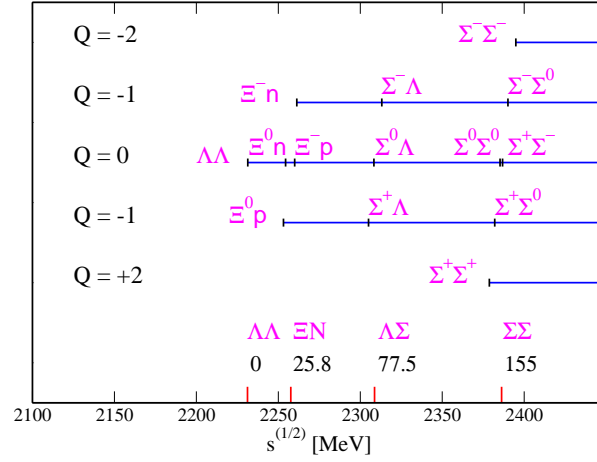


FIG. 1: Graphic representation of the thresholds for the various strangeness $S = -2$ channels with charges from $Q = -2$ to $Q = +2$. The thresholds for isospin averaged masses are indicated at the bottom.

Moreover, no polarization experiment is required because (near threshold) the $\Lambda\Lambda$ can be only in the (spin singlet) 1S_0 partial wave. The 3S_1 state is forbidden due to the Pauli principle, as already mentioned. Thus, spin triplet states can only occur in P (or higher partial) waves - and it is safe to assume that such higher partial waves do not contribute near threshold. There is, however, a complication because the first inelastic threshold (due to ΞN) is fairly close: the $\Xi^0 n$ channel opens at an excess energy of 23.06 MeV, c.f. Fig. 1. (In the Λp case considered in [28] the first inelastic channel ($\Sigma^0 p$) opens at 76.96 MeV!) Thus, it is necessary to re-do the error estimate of Ref. [28]. This will be done in subsection A below. Note that, for convenience, we will work with isospin-averaged masses throughout this section so that the ΞN threshold is located at 25.8 MeV!

In subsection B we apply our method to the $\Lambda\Lambda$ invariant mass distribution measured in the reaction $^{12}C(K^-, K^+ \Lambda \Lambda X)$ by the KEK-PS E224 Collaboration [8]. Those data, though afflicted by sizeable error bars, allow us to demonstrate how our method works, and they even enable us to deduce a concrete value for the $\Lambda\Lambda$ 1S_0 scattering length.

A. Error estimation

In this subsection, we generalize the discussion of theoretical errors presented in [28] to the case of the occurrence of inelastic channels. Specifically, we estimate the theoretical error for the extraction of the hyperon-hyperon scattering length exemplary for the reaction $K^- d \rightarrow K^0 \Lambda \Lambda$ taking into account that there is a near-by inelastic threshold due to the coupling of $\Lambda\Lambda$ to the ΞN channel.

The uncertainties originate [28] from the energy (i.e. m^2) dependence of the function $\Phi(s, t, m^2)$ in Eq. (1). These include the energy dependence of the production operator (i.e. the influence of left-hand singularities), contributions of the elastic scattering to the dispersion integral at higher energies, the influence of inelastic channels, and the interaction between other pairs of particles in the final state. The latter effect can be controlled by choosing different kinematical conditions such as initial energy (final-state interaction among other pairs of particles would depend on such a choice whereas the $\Lambda\Lambda$ FSI does not). Also investigating the invariant-mass distribution for a corresponding pair of particles via a Dalitz plot analysis can provide additional information on their interaction [43]. In what follows, we will disregard this (possibly important) kind of correction and focus on the other three.

The energy dependence of the production amplitude $A_S(s, t, m^2)$ can be deduced from the basic principles of analyticity and unitarity. The discontinuity of the amplitude in m^2 is given by the sum of the elastic term, the inelastic contribution of the ΞN (and/or other) channel, $D_S^{in}(s, t, m^2)$, and the left-hand part, $D_S^{l.h.}(s, t, m^2)$, denoting the remaining contribution from the production operator

$$D_S(s, t, m^2) \equiv \frac{1}{2i}(A_S(s, t, m^2 + i0) - A_S(s, t, m^2 - i0)) = A(s, t, m^2)e^{-i\delta} \sin \delta + D_S^{l.h.}(s, t, m^2) + D_S^{in}(s, t, m^2). \quad (6)$$

The inelastic contribution reads

$$D_S^{in}(s, t, m^2) = \left(\frac{A(s, t, m^2)(1 - \eta)e^{-2i\delta}}{2i} + A_2(s, t, m^2)f_{12}^*(m^2)p_2 \right) \theta(m^2 - m_2^2), \quad m_2^2 = (m_\Xi + m_N)^2, \\ p_2 = \sqrt{\frac{(m^2 - m_2^2)m_\Xi m_N}{m_2^2}} \approx \frac{1}{2}\sqrt{m^2 - m_2^2} \quad (\text{in non-relativistic kinematics}), \quad (7)$$

where η is the inelasticity parameter in the $\Lambda\Lambda$ system, A_2 is the production amplitude for the reaction $K^-d \rightarrow K^+\Xi N$, and f_{12} is the $\Lambda\Lambda \rightarrow \Xi N$ transition amplitude. The latter can be written in terms of η , δ , and the phase shift δ_2 of the ΞN channel:

$$f_{12} = \frac{\sqrt{1 - \eta^2} e^{i(\delta + \delta_2)}}{2\sqrt{p} p_2}. \quad (8)$$

In order to shorten the notation we rewrite D_S^{in} as

$$D_S^{in}(m^2) = A(m^2)\theta(m^2 - m_2^2) \left(\frac{1 - \eta}{2i} e^{-2i\delta} + \left| \frac{A_2(m^2)}{A(m^2)} \right| \frac{|\tilde{f}_{12}(m^2)|}{2} e^{-i\tilde{\delta}} \right), \\ \tilde{f}_{12}(m^2) = 2p_2 f_{12}(m^2), \quad \tilde{\delta} = \delta + \delta_2 + \delta_A - \delta_{A_2}, \quad (9)$$

where we suppressed any dependence on s and t , and where we denoted the phases of the production amplitudes A and A_2 by δ_A and δ_{A_2} , respectively. The solution of Eq. (6) in the physical region can be represented as (see Refs. [28, 37, 38, 44])

$$A(m^2) = e^{u(m^2)} \tilde{\Phi}(m^2) \equiv e^{u(m^2)} (\Phi_{l.h.}(m^2) + \Phi_{in}(m^2)), \\ \Phi_{l.h.}(m^2) = \int_{-\infty}^{\tilde{m}^2} \frac{D_S^{l.h.}(m'^2) e^{-u(m'^2)} dm'^2}{m'^2 - m^2 - i0} \frac{1}{\pi}, \quad \Phi_{in}(m^2) = \int_{m_2^2}^{+\infty} \frac{D_S^{in}(m'^2) e^{-u(m'^2)} (m^2 - m_0^2) dm'^2}{(m'^2 - m^2 - i0)(m'^2 - m_0^2)} \frac{1}{\pi}, \\ u(m^2) = \frac{1}{\pi} \int_{m_0^2}^{\infty} \frac{\delta(m'^2)(m^2 - m_0^2)}{(m'^2 - m^2 - i0)(m'^2 - m_0^2)} dm'^2. \quad (10)$$

where \tilde{m}^2 denotes the upper end of the left-hand cut. In order to remove the energy independent part from the inelastic dispersion integral, we made subtractions at $m^2 = m_0^2$ in the definition of $u(m^2)$ and in the inelastic dispersion integral (a constant term is assigned then to the left-hand contribution which anyway, as we will show, is slowly varying with energy.)

The theoretical error of the extracted scattering length is determined by the energy dependence of the function $\Phi(s, t, m^2)$ from Eq. (1) [28]

$$\delta a^{(th)} = - \lim_{m^2 \rightarrow m_0^2} \mathbf{P} \int_{m_0^2}^{m_{max}^2} \frac{\log |\Phi(m'^2)|^2}{\sqrt{m'^2 - m_0^2} (m'^2 - m^2)} \sqrt{\frac{m_{max}^2 - m^2}{m_{max}^2 - m'^2}} \frac{dm'^2}{\pi}. \quad (11)$$

The function $\Phi(m^2)$ depends on m_{max}^2 . This dependence factors out [28] $\Phi(m_{max}^2, m^2) = \Psi(m_{max}^2, m^2) \tilde{\Phi}(m^2)$ where $\Psi(m_{max}^2, m^2)$ contains the information on the phase shift at energies above m_{max}

$$\Psi(m_{max}^2, m^2) \propto \exp \left[\frac{1}{\pi} \int_{m_{max}^2}^{\infty} \frac{\delta(m'^2)}{m'^2 - m^2 - i0} dm'^2 \right]. \quad (12)$$

Thus the theoretical error is the sum $\delta a^{(th)} = \delta a^{m_{max}} + \delta \tilde{a}$, where $\delta a^{m_{max}}$ is due to the factor $\Psi(m_{max}^2, m^2)$ and $\delta \tilde{a}$ is determined by the energy dependence of $\tilde{\Phi}(m^2)$ which is related to the energy dependence of the production operator and inelastic effects.

Let us first estimate the theoretical error originating from the energy dependence of the production operator, neglecting for the moment the inelastic contributions. In order to do this one has to investigate the contribution to the dispersion integral from the left-hand cuts. We follow here the procedure utilized in [28]. We are interested in the left-hand singularities of the amplitude, i.e. singularities in some momentum transfer variable t . The simplest production mechanism for the reaction $K^-d \rightarrow K^0 \Lambda \Lambda$ is seemingly the one shown in Fig. 2 denoting the exchange

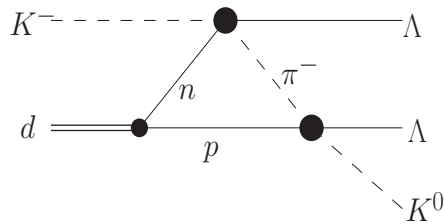


FIG. 2: Typical production mechanism for the reaction $K^- d \rightarrow K^0 \Lambda \Lambda$.

of one nucleon and one pion (one should add, of course, the diagram with proton and nucleon interchanged and π^+ replaced by π^0). Clearly, there are also other more complicated production mechanisms that will contribute. However, those should be of even shorter range and thus, correspond to production operators with even weaker energy dependence so that they are not relevant for the estimation of the theoretical error. We consider for simplicity kinematics corresponding to the final particles produced not far away from threshold (so that the momenta of the final particles are small compared to the momentum transfer). The required initial CM momentum is then $p_i \approx 580$ MeV/c. The production operator contains a cut corresponding to the $\pi^- p$ intermediate state, which can be associated with the interaction in the ΛK^0 system discussed above. The other cut over the neutron and π^- is the one we are interested in. We can very roughly estimate the energy dependence associated with this singularity via approximating it by a pole term,

$$\Phi(m^2) \sim \frac{1}{t - m_{n\pi^-}^2}, \quad t = (p_{K^-} - p_\Lambda)^2, \quad (13)$$

where p_{K^-} and p_Λ are the corresponding 4-momenta and $m_{n\pi^-}$ is the effective invariant mass of the $n\pi^-$ system, i.e. $m_{n\pi^-} \approx 1$ GeV. For threshold kinematics (i.e. zero momentum of all final particles – this choice is made for definiteness) this energy dependence (after averaging over all directions) has the form $\Phi \approx 1 + p^2/p_t^2$, where p is the CM momentum in the $\Lambda\Lambda$ system and p_t is of the order of 2 GeV. The correction to the scattering length due to such an energy dependence of the amplitude amounts to [28] $\frac{p_{max}}{p_t^2} \approx 0.01$ fm for values of $\epsilon_{max} = 40 - 60$ MeV. The above rough estimation is sufficient to observe that the error coming from the energy dependence of the production operator is negligible as compared to the other sources of uncertainties and we can safely assume that

$$\Phi_{l.h.}(m^2) \approx \Phi_{l.h.}(m_0^2) = A(m_0^2) \approx A(m^2)e^{-u(m^2)}. \quad (14)$$

Note that since the $\Lambda\Lambda$ scattering length is expected to be somewhat smaller than $a_{\Lambda p}$ [16, 19, 22, 24] we choose $\epsilon_{max} = 60$ MeV rather than $\epsilon_{max} = 40$ MeV (used for ΛN scattering) as our central value in order to minimize the effect of higher energy scattering. As was pointed out in Ref. [28] ϵ_{max} must be chosen well above $\frac{1}{a_S^2 m_\Lambda}$ (cf. also the discussion in Sect. II). Although a further increase of ϵ_{max} can help even more in reducing this effect, in reality it would be difficult in this case to separate S -waves from higher partial waves in the final state and to avoid the influence of the interaction in other channels – the effects and the number of inelastic channels would increase.

Next we consider the error coming from the inelastic channels coupled to the $\Lambda\Lambda$ system. For simplicity we consider only the one nearest to the $\Lambda\Lambda$ threshold, namely ΞN , which opens only about 25 MeV above the $\Lambda\Lambda$ threshold so that it is necessary to analyze its impact. Clearly, due to the lack of empirical information on the ΞN interaction and the coupling of this channel to the $\Lambda\Lambda$ system such an error analysis cannot be done in a completely model-independent way. One has to make some assumptions on the strength of the interactions in the relevant channels. For our analysis we prepared three variants of the hyperon-hyperon interaction of [24] which yield $\Lambda\Lambda$ scattering lengths of -1.36 fm, -1.50 fm, and -1.70 fm, respectively. We are interested in the situation when the effect coming from the inelastic channel is small and its contribution can be treated perturbatively, i.e.

$$\Phi_{in}(m^2) \ll \Phi_{l.h.}(m^2). \quad (15)$$

If this is not the case the error of the extraction of the scattering length would be comparable with the scattering length itself and then an extraction would no longer be meaningful. We made a subtraction in Eq. (10) at $m^2 = m_0^2$ so that Φ_{in} is exactly zero at the beginning of the integration interval. Then a small resulting correction to the scattering length would imply a likewise small variation of Φ_{in} so that then the condition (15) would be justified. Note that formally the subtraction point does not enter the expression for the error.

Utilizing Eqs. (15) and (14) one can rewrite the formula (11) for the inelastic contribution to the theoretical error in the form

$$\begin{aligned}
\delta a^{in} &= - \lim_{m^2 \rightarrow m_0^2} \frac{1}{\pi} \mathbf{P} \int_{m_0^2}^{m_{max}^2} \frac{\log |\Phi_{l.h.}(m'^2) + \Phi_{in}(m'^2)|^2}{\sqrt{m'^2 - m_0^2} (m'^2 - m^2)} \sqrt{\frac{m_{max}^2 - m^2}{m_{max}^2 - m'^2}} dm'^2 \\
&\approx - \lim_{m^2 \rightarrow m_0^2} \frac{1}{\pi} \mathbf{P} \int_{m_0^2}^{m_{max}^2} \frac{2\Re\{\Phi_{in}(m'^2)/A(m_0^2)\}}{\sqrt{m'^2 - m_0^2} (m'^2 - m^2)} \sqrt{\frac{m_{max}^2 - m^2}{m_{max}^2 - m'^2}} dm'^2 \\
&= - \frac{1}{\pi} \int_{m_0^2}^{m_{max}^2} \frac{2\Re\{(\Phi_{in}(m'^2) - \Phi_{in}(m_0^2))/A(m_0^2)\}}{\sqrt{m'^2 - m_0^2} (m'^2 - m_0^2)} \sqrt{\frac{m_{max}^2 - m_0^2}{m_{max}^2 - m'^2}} dm'^2.
\end{aligned} \tag{16}$$

Substituting $\Phi_{in}(m^2)$ from Eq. (10) and performing one integration one gets

$$\begin{aligned}
\delta a^{in} &= \Im \frac{4p_{max}}{\pi} \int_{m_0^2}^{m_{max}^2} dm^2 \frac{D_S^{in}(m^2) e^{-u(m^2)}}{A(m_0^2) (m^2 - m_0^2)^{\frac{3}{2}} \sqrt{m_{max}^2 - m^2}} \\
&\quad - \Re \frac{4p_{max}}{\pi} \int_{m_0^2}^{m_{max}^2} dm^2 \frac{D_S^{in}(m^2) e^{-u(m^2)}}{A(m_0^2) (m^2 - m_0^2)^{\frac{3}{2}} \sqrt{m^2 - m_{max}^2}},
\end{aligned} \tag{17}$$

where $p_{max} = \frac{1}{2} \sqrt{m_{max}^2 - m_0^2}$.

Using the explicit form of $D_S^{in}(m^2)$ from Eq. (7) and making a suitable change of variables we have

$$\begin{aligned}
\delta a^{in} &= \delta a_1^{in} + \delta a_2^{in}, \\
\delta a_1^{in} &= \frac{1}{\pi p_{max}} \left(\int_0^{y_2} \frac{d\tilde{y}}{(1 - \tilde{y}^2)^{(3/2)}} (\eta - 1) \cos(2\delta) + \int_0^\infty \frac{dy}{(1 + y^2)^{(3/2)}} (1 - \eta) \sin(2\delta) \right), \\
\delta a_2^{in} &= - \frac{1}{\pi p_{max}} \left(\int_0^{y_2} \frac{d\tilde{y}}{(1 - \tilde{y}^2)^{(3/2)}} |\tilde{f}_{12}| \sin \tilde{\delta} \left| \frac{A_2}{A} \right| + \int_0^\infty \frac{dy}{(1 + y^2)^{(3/2)}} |\tilde{f}_{12}| \cos \tilde{\delta} \left| \frac{A_2}{A} \right| \right),
\end{aligned} \tag{18}$$

with $\tilde{y} = \sqrt{\frac{m_{max}^2 - m^2}{m_{max}^2 - m_0^2}}$, $y_2 = \sqrt{\frac{m_{max}^2 - m_0^2}{m_{max}^2 - m_0^2}}$, and $y = \sqrt{\frac{m^2 - m_{max}^2}{m_{max}^2 - m_0^2}}$. From [28] we recall the expression for $\delta a^{m_{max}}$ (with the same definition of y)

$$\delta a^{m_{max}} = \frac{2}{\pi p_{max}} \int_0^\infty \frac{\delta(y) dy}{(1 + y^2)^{(3/2)}}. \tag{19}$$

Note that the integration over y from 0 to ∞ can be truncated at $y = 1$, say, for practical applications since the integrals are rapidly converging (unless the phase shift is rising unnaturally fast with energy). In any case, we cannot trust the χ EFT predictions for the amplitude at such high energies.

The numerical values of $\tilde{\delta}$ and of $\left| \frac{A_2}{A} \right|$ are not known, therefore we estimate

$$|\delta a_2^{in}| < \frac{1}{\pi p_{max}} \left(\int_0^{y_2} \frac{d\tilde{y}}{(1 - \tilde{y}^2)^{(3/2)}} |\tilde{f}_{12}| + \int_0^\infty \frac{dy}{(1 + y^2)^{(3/2)}} |\tilde{f}_{12}| \right) \text{Max} \left| \frac{A_2}{A} \right|, \tag{20}$$

where $\text{Max} \left| \frac{A_2}{A} \right|$ is the maximal value of this ratio in the considered energy region.

Using the three mentioned variants of the hyperon-hyperon interaction we arrive at the following estimates of the theoretical errors for $\epsilon_{max} = 60$ MeV:

$$\delta a_1^{in} + \delta a^{m_{max}} = -0.19 \text{ fm}, |\delta a_2^{in}| < 0.28 \text{Max} \left| \frac{A_2}{A} \right| \text{ fm},$$

for the variant with $a = -1.36$ fm,

$$\delta a_1^{in} + \delta a^{m_{max}} = -0.11 \text{ fm}, |\delta a_2^{in}| < 0.30 \text{Max} \left| \frac{A_2}{A} \right| \text{ fm},$$

for the variant with $a = -1.50$ fm,

$$\delta a_1^{in} + \delta a^{m_{max}} = -0.22 \text{ fm}, |\delta a_2^{in}| < 0.14 \text{Max} \left| \frac{A_2}{A} \right| \text{ fm},$$

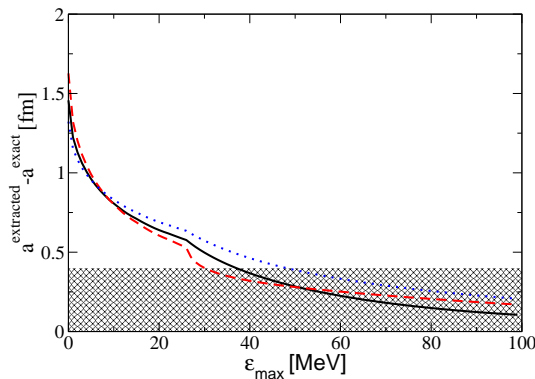


FIG. 3: Dependence of the extracted scattering lengths on the value of the upper limit of integration, ϵ_{max} . Shown is the difference to the exact results for three variants of the χ EFT $\Lambda\Lambda$ interaction with $a = -1.50$ fm (solid line), $a = -1.70$ fm (dashed line), and $a = -1.36$ fm (dotted line). The shaded area indicates the estimated error of the applied method.

for the variant with $a = -1.70$ fm. The value of $\text{Max} \left| \frac{A_2}{A} \right|$ can only be accessed from the corresponding production experiment for the ΞN channel. Some estimates can be obtained by looking at the strength of the cusp effect in the $\Lambda\Lambda$ production channel. Under the assumption that there is no specific production mechanism that makes this ratio large, one can estimate the ratio, at least qualitatively, from the unitarity contribution that correspond to the (on shell) $\Lambda\Lambda \rightarrow \Xi N$ conversion. It amounts to $\text{Max} \left| \frac{A_2}{A} \right| = \text{Max} |f_{12} p_2| < 0.3$ for all three considered interactions. Based on those numbers a rough estimation for the full theoretical error related to inelastic effects and higher energy scattering yields $\delta a^{th} < 0.3 - 0.4$ fm. Such small values for δa^{th} justify the approximation made in Eq. (15), because it means that $\Phi_{in}(m^2)$ does not change much for $p < \frac{1}{\delta a^{th}}$, an energy range that safely covers the region we are interested in.

In order to test our method and to check our error analysis we applied it to a production amplitude, calculated with a point-like production operator, that incorporates $\Lambda\Lambda$ final-state interactions generated from the three variants of the χ EFT interaction. In Fig. 3 we show the dependence of the difference of the extracted scattering length and the exact one on the cut off ϵ_{max} of the integration. One can see that for $\epsilon_{max} = 60$ MeV the theoretical error is indeed within the range of $0.25 - 0.35$ fm in agreement with the preceding analysis. Note the apparent drop of the curves around 25 MeV, i.e. at the opening of the ΞN channel.

B. Analysis of data on the $\Lambda\Lambda$ invariant mass from $^{12}C(K^-, K^+ \Lambda\Lambda X)$

First results for the $\Lambda\Lambda$ invariant mass distribution were reported by the KEK-PS E224 Collaboration from a measurement of the $^{12}C(K^-, K^+ \Lambda\Lambda X)$ reaction in 1998 [31]. An enhancement was seen for invariant masses near threshold. Already at that time there were attempts to extract the $\Lambda\Lambda$ interaction from the spectrum [45, 46]. Then in 2007 the KEK-PS E522 Collaboration published a $\Lambda\Lambda$ invariant mass spectrum with somewhat better statistics [8]. Also in this case efforts were made to extract the $\Lambda\Lambda$ scattering length. The value reported at some conferences [47, 48], $a_{\Lambda\Lambda} = -0.10_{-1.56}^{+0.37} \pm 0.28$ fm, was obtained by utilizing the Migdal-Watson approach (Eq. (5)).

As already pointed out above and as we thoroughly investigated in [29], the Migdal-Watson approach works only well for fairly large scattering lengths, i.e. for values of the order of 5 fm or more, as they are typical for the NN interaction. For small scattering lengths as suggested by the analysis in [47, 48] this approach is not reliable. It can lead to a systematic deviation of 0.7 fm or more. Thus, we re-analyzed the $\Lambda\Lambda$ invariant mass spectrum given in [8], employing our method based on Eq. (2). Indeed, since in the $\Lambda\Lambda$ case the FSI can only occur in the 1S_0 partial wave and, thus, no polarization experiment is required, our method can be applied also to data like those of $\Lambda\Lambda$ production on carbon [8] from the reaction $^{12}C(K^-, K^+ \Lambda\Lambda X)$. But one has to keep in mind that in reactions on nuclei the energy dependence of the production operator is not so well under control. For example, there could be excitations in the other fragments of the reaction process. In addition, in the concrete case, the error bars of the $\Lambda\Lambda$ invariant mass distribution are quite large, therefore one has to expect large uncertainties for the extracted scattering length. Nevertheless, in order to demonstrate how the method works we applied it to the data of Ref. [8], following the procedure for the analysis of experimental data described in detail in the Appendix A of [28]. We fit the data with the amplitude squared parametrized as

$$|A(m)|^2 = \exp \left[C_0 + \frac{C_1^2}{(m^2 - C_2^2)} \right], \quad (21)$$

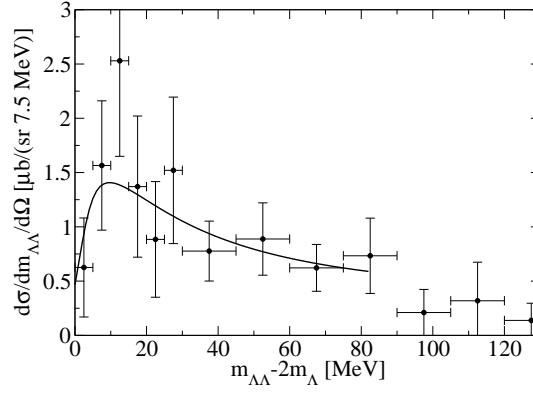


FIG. 4: $\Lambda\Lambda$ invariant mass spectrum for the reaction $^{12}\text{C}(K^-, K^+\Lambda\Lambda X)$ [8] and our fit to it (solid line)

multiplied with the phase-space factor, and allowing for a finite mass resolution of 2.5 MeV. The resulting curve is shown in Fig. 4 (solid line). Then we use this fit to extract the scattering length from the dispersion integral with the cut off $\epsilon_{max} = 60$ MeV. The result is

$$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \pm 0.4 \text{ fm}, \quad (22)$$

where the first error is due to the uncertainties in the data and the second value is the theoretical error estimated in the preceding subsection.

TABLE I: $\Lambda\Lambda$ 1S_0 scattering lengths (a_s) and effective range parameters (r_s) for various strangeness $S = -2$ interaction potentials (in fm). In case of the χ EFT interaction results for the lowest (550 MeV) and highest (700 MeV) cut-off value are given, cf. [24]. Note that (a) the scattering lengths of the Nijmegen (ESC04) potential differ significantly depending on whether they are calculated in particle [17] or isospin [18] basis, (b) in the potentials by Tominaga et al. [20] some channel couplings are not included.

YY interaction	reference	a_s [fm]	r_s [fm]
χ EFT (550)	[24]	-1.52	0.82
χ EFT (700)	[24]	-1.67	0.34
Nijmegen (NSC97a)	[16]	-0.27	15.00
Nijmegen (NSC97f)	[16]	-0.35	14.68
Nijmegen (ESC04a)	[18]	-3.804	2.42
Nijmegen (ESC04d)	[18]	-1.555	3.62
Nijmegen (ESC08a'')	[19]	-0.88	4.34
Tominaga (set B)	[20]	-3.40	2.79
Fujiwara (fss2)	[22]	-0.821	3.78
Valcarce	[23]	-2.54	-

For the ease of comparison we present in Table I a selection of $\Lambda\Lambda$ 1S_0 scattering lengths predicted by various published YY interaction models. The large scattering length of the interaction by Tominaga et al. from 1998 [20] was still triggered by the first experimental information on the ground states of ${}_{\Lambda\Lambda}^6\text{He}$, ${}_{\Lambda\Lambda}^{10}\text{Be}$, and ${}_{\Lambda\Lambda}^{13}\text{B}$ [2–4]. Those experiments suggested a separation energy of $\Delta B_{\Lambda\Lambda} = 4 - 5$ MeV, where the separation energy is defined as, e.g., $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) - 2B_{\Lambda}({}_{\Lambda}^5\text{He})$. Such a large separation energy could be only described with a rather strong $\Lambda\Lambda$ interaction with scattering lengths $a_{\Lambda\Lambda}$ in the order of -2 to -3.6 fm [20, 49] or larger [50].

The analysis of the unambiguously identified ${}_{\Lambda\Lambda}^6\text{He}$ double hypernucleus (Nagara event) [5] yielded the much smaller separation energy of 1.01 ± 0.20 MeV. Calculations that obtain separation energies in agreement with the new experimental value suggest $\Lambda\Lambda$ scattering lengths in the order of -0.7 to -1.3 fm [17, 22, 50]. It should be said, however, that so far there are no fully microscopic (i.e. six-body) calculations of ${}_{\Lambda\Lambda}^6\text{He}$ available that utilize only elementary baryon-baryon (NN , YN , YY) interactions such as those listed in Table I. All studies are performed either with three-body Faddeev equations applied to the cluster model [50–54], the Brueckner theory approach [17, 55], or with the stochastic variational method [56], and rely, at least partly, on effective two-body interactions.

In this context let us mention that the separation energy for the Nagara event has been recently re-analysed and is now given as $\Delta B_{\Lambda\Lambda} = 0.67 \pm 0.17$ MeV [57]. See also Ref. [58] for a recent review of the status of $\Lambda\Lambda$ hypernuclei.

TABLE II: $\Sigma\Sigma$ and ΞN S -wave scattering lengths a and effective range parameters r for various strangeness $S = -2$ interaction potentials (in fm). The subscripts s and t refer to the singlet (1S_0) and triplet (3S_1) states, respectively. In case of the χ EFT interaction results for the lowest (550 MeV) and highest (700 MeV) cut-off value are given, cf. [24]. Note that (a) the scattering lengths of the Nijmegen (ESC04) potential differ significantly depending on whether they are calculated in particle [17] or isospin [18] basis, (b) in the potentials [20, 21] some channel couplings are not included.

YY interaction	reference	channel	a_s [fm]	r_s [fm]	a_t [fm]	r_t [fm]
χ EFT (550)	[24]	$\Sigma^+\Sigma^+$	-6.23	2.17	-	-
χ EFT (700)	[24]	$\Sigma^+\Sigma^+$	-9.27	1.88	-	-
Nijmegen (NSC97a)	[16]	$\Sigma^+\Sigma^+$	10.32	1.60	-	-
Nijmegen (NSC97f)	[16]	$\Sigma^+\Sigma^+$	6.98	1.46	-	-
Fujiwara (fss2)	[22]	$\Sigma^+\Sigma^+$	-85.3	2.34	-	-
Valcarce	[23]	$\Sigma^+\Sigma^+$	0.523		-	-
χ EFT (550)	[24]	$\Xi^0 n$	-	-	-0.34	-5.86
χ EFT (700)	[24]	$\Xi^0 n$	-	-	-0.15	16.3
Nijmegen (ESC04a)	[18]	$\Xi N(I=0)$	-	-	-1.672	2.70
Nijmegen (ESC04d)	[18]	$\Xi N(I=0)$	-	-	122.5	2.083
Nijmegen (ESC08a'')	[19]	$\Xi N(I=0)$	-	-	6.9	1.18
Tominaga (set B)	[20]	$\Xi N(I=0)$	-	-	-0.352	17.4
Ehime (1.82)	[21]	$\Xi N(I=0)$	-	-	-0.43	13.0
Valcarce	[23]	$\Xi N(I=0)$	-	-	0.28	
χ EFT (550)	[24]	$\Xi^- n$	0.21	-30.7	0.02	968
χ EFT (700)	[24]	$\Xi^- n$	0.13	-98.5	0.03	548
Nijmegen (NSC97a)	[16]	$\Xi^- n$	0.46	-6.09	-0.04	634
Nijmegen (NSC97f)	[16]	$\Xi^- n$	0.40	-8.88	-0.31	870
Nijmegen (ESC04a)	[18]	$\Xi^- n$	0.491	-0.421		
Nijmegen (ESC04d)	[18]	$\Xi^- n$	0.144	4.670		
Nijmegen (ESC08a'')	[19]	$\Xi^- n$	0.58	-2.71	3.49	0.60
Tominaga (set B)	[20]	$\Xi^- n$	-0.202	33.0	-0.484	10.6
Ehime (1.82)	[21]	$\Xi^- n$	-0.27	20.3	-0.56	9.0
Fujiwara (fss2)	[22]	$\Xi^- n$	0.324	-8.93	-0.207	26.2
Valcarce	[23]	$\Xi^0 p$	-3.32		18.69	

IV. THE ΞN AND $\Sigma\Sigma$ SCATTERING LENGTHS

Since the $\Lambda\Lambda$ system is a pure isospin $I = 0$ state, the $I = 1$ ΞN interaction is also elastic and, thus, permits a determination of the corresponding scattering length via our method. The first inelastic channel, $\Lambda\Sigma$, opens at the excess energy of around 52 MeV and, therefore, should affect the extraction of the scattering length less than what has been discussed in the context of the $\Lambda\Lambda$ case above. The required ΞN invariant mass spectrum is accessible experimentally in the reaction $K^- d \rightarrow K^+ \Xi^- n$, for example. However, since ΞN can occur in the 1S_0 as well as in the 3S_1 partial wave, one needs data from an experiment with polarization for a separation of the singlet and triplet contributions. The relevant observables are discussed in Appendix A. In addition, one has to keep in mind that the reaction $K^- d \rightarrow K^+ \Xi N$ might be dominated by the quasi-elastic process $K^- N \rightarrow K^+ \Xi$, similar to what happened for the reaction $\gamma d \rightarrow KY N$ [30] with $\gamma N \rightarrow KY$. In such a case the reaction kinematics has to be chosen rather carefully in order to suppress the contributions from the quasi-elastic process, as studied in detail in [30], which certainly increases the difficulties for a corresponding experiment.

As a subtlety let us mention that even in the $\Xi^0 n$ system the scattering length for 3S_1 is real, although the amplitude is actually the sum of $I = 0$ and $I = 1$ states. Because of the Pauli principle, the 3S_1 partial wave of the $\Lambda\Lambda$ system is forbidden so that there is no coupling between the $\Lambda\Lambda$ and ΞN channels for the partial wave in question.

Experimental information on the ΞN invariant mass spectrum is scarce [59–61]. Results published in Ref. [59] for the $\Xi^- p$ case, obtained from a $K^- d$ bubble-chamber experiment, suggest an enhancement in the invariant mass distribution - but at around 2480 MeV and not near the ΞN threshold which is at around 2255 MeV. In any case, the statistics is too low for drawing any conclusions. The situation is better for an experiment performed at Saclay [60] where the missing mass (MM) in the reaction $K^- d \rightarrow K^+ + MM$ at 1.4 GeV/c was studied. The curve presented in this publication exhibits a rather smooth behaviour around the ΞN threshold which suggests that the $\Xi^- n$ interaction in the 1S_0 and/or 3S_1 might be fairly weak. Such a conjecture is actually in line with the results of several of the potential models summarized in Table II which predict rather small $\Xi^- n$ scattering lengths. We want to emphasize, however, that there are also models with a fairly strong ΞN interaction. Notably the latest Nijmegen potential (ESC08a”) produces bound states in the 3S_1 partial wave of the $I = 0$ and $I = 1$ channels [19]. The binding energies are comparable to that of the deuteron and, accordingly, sizeable near-threshold enhancements in the corresponding invariant mass spectrum are to be expected if such bound states indeed exist in nature.

Besides the ΞN system the $I = 2$ channel $\Sigma^+ \Sigma^+$ is potentially interesting too because it is also elastic. But due to the charge it cannot be produced with a K^- beam on the deuteron. However, the $\Sigma^- \Sigma^-$ system which is likewise $I = 2$ could be studied, namely in the reaction $K^- d \rightarrow K^+ \pi^- \Sigma^- \Sigma^-$. Also in this case, only the 1S_0 partial wave is present so that no polarization data are required for a determination of the scattering length. Contrary to $\Xi^- n$, here practically all model predictions for the $\Sigma^+ \Sigma^+$ ($\Sigma^- \Sigma^-$) scattering length are fairly large, cf. Table II. Details of the application of our method to cases where the Coulomb interaction is present can be found in Ref. [29].

In principle, one can even consider reactions of the type $K^- d \rightarrow KK\Xi\Lambda$ and $K^- d \rightarrow KK\Xi\Sigma$, which would give access to the strangeness $S = -3$ world. Potential-model calculations [17, 22] and also predictions obtained within the framework of χ EFT [62] suggest that the interaction in some of the channels are strongly attractive so that the corresponding scattering lengths could be large. Clearly, here it would be desirable to have at least a rough estimation of the count rates that one can expect in order to judge the feasibility of such experiments. Independently of that, also for these reactions we provide and discuss the relevant polarization observables that are needed for a separation of the singlet and triplet contributions, cf. Appendix A.

V. SUMMARY

We reviewed a method that allows one to extract hadronic scattering lengths from production reactions by studying final-state interactions. In particular, we discussed its applicability to the case of baryon-baryon interactions in the strangeness $S = -2$ and $S = -3$ sectors. We emphasized the importance of separating different spin states of the interacting particles. Considering as examples the reactions $K^- d \rightarrow KB_1B_2$, $\gamma d \rightarrow K_1K_2B_1B_2$ and $K^- d \rightarrow K_1K_2B_1B_2$, we could demonstrate that it is possible to construct polarization observables that provide access to spin-singlet and spin-triplet scattering lengths. In case of the $\Lambda\Lambda$ and $\Sigma^+ \Sigma^+$ (or $\Sigma^- \Sigma^-$) interactions near threshold only the 1S_0 partial wave is present due to the Pauli principle and, thus, no polarization experiments are required for determining the pertinent scattering length from the final-state interaction. Employing the method to available data on the $\Lambda\Lambda$ invariant mass from $^{12}C(K^-, K^+ \Lambda\Lambda X)$ [8], a 1S_0 scattering length of $a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm is deduced. The error given here reflects the accuracy of those data. Thus, it would be important to perform experiments with better statistics which could be done, e.g., at J-PARC [63]. This would then allow one to reduce the error on the $\Lambda\Lambda$ scattering length to the one of the extraction method, which we estimate to be in the order of 0.3 – 0.4 fm.

Acknowledgments

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Appendix A: Spin considerations for the production of baryon-baryon systems with strangeness $S = -2$ and $S = -3$

The technique utilized for the hyperon-nucleon interactions in [28–30] is applicable also in the case of strangeness $S = -2$ and $S = -3$ systems. The necessary condition that baryon-baryon scattering should be elastic up to some $m = m_{max}$ is satisfied for the following $S = -2$ channels: $\Lambda\Lambda$, $\Sigma^+ \Sigma^+$, $\Sigma^- \Sigma^-$, $\Xi^0 p$, $\Xi^- n$, and for the $S = -3$ channels: $\Xi^- \Lambda$, $\Xi^0 \Lambda$, $\Xi^- \Sigma^-$, $\Xi^0 \Sigma^+$. See Fig. 1 for a graphic overview of the kinematics. In what follows we are going to consider as examples $K^- d$, pp , and γd scattering in complete analogy with the hyperon-nucleon production reactions studied

in [30]. The following four types of reactions can be used to produce baryon-baryon states with strangeness $S = -2$ and $S = -3$:

$$K^-d \rightarrow KB_1B_2 \quad (K^0\Lambda\Lambda, K^+\Xi^-n), \quad (\text{A1})$$

$$pp \rightarrow K_1K_2B_1B_2 \quad (K^+K^+\Lambda\Lambda, K^+K^0\Xi^0p, K^0K^0\Sigma^+\Sigma^+), \quad (\text{A2})$$

$$\gamma d \rightarrow K_1K_2B_1B_2 \quad (K^+K^0\Lambda\Lambda, K^0K^0\Xi^0p, K^+K^+\Xi^-n), \quad (\text{A3})$$

$$K^-d \rightarrow K_1K_2B_1B_2 \quad (K^0K^0\Xi^0\Lambda, K^+K^0\Xi^-\Lambda, K^+K^+\Xi^-\Sigma^-). \quad (\text{A4})$$

Now we come to the question of separating the spin-triplet and spin-singlet states in the baryon-baryon system. As in the case of the hyperon-nucleon interaction it is sufficient to consider reactions with polarized initial particles. We start from the general form for the reaction amplitude in the center-of-mass (CM) system for the three processes

$$\begin{aligned} \mathcal{M}_{K^-d \rightarrow KB_1B_2} &= a_1^s(\vec{\epsilon}_d \times \hat{p}) \cdot \vec{k} + a_2^t(\vec{\epsilon}_d \cdot \vec{S}') + a_3^t(\vec{\epsilon}_d \cdot \hat{p})(\vec{S}' \cdot \vec{k}) + a_4^t(\vec{\epsilon}_d \cdot \vec{k})(\vec{S}' \cdot \hat{p}), \\ \mathcal{M}_{pp \rightarrow K_1K_2B_1B_2} &= b_1^s + b_2^s(\hat{p} \times \hat{k}) \cdot \vec{S} + b_3^t(\hat{p} \cdot \hat{k})(\hat{p} \times \hat{k}) \cdot \vec{S}' + (b_4^t\hat{p}_i\hat{k}_j + b_5^t\hat{p}_j\hat{k}_i)S_iS'_j, \\ \mathcal{M}_{\gamma d \rightarrow K_1K_2B_1B_2} &= c_1^s(\vec{\epsilon}_\gamma \times \vec{\epsilon}_d) \cdot \hat{p} + c_2^t(\vec{\epsilon}_\gamma \cdot \vec{\epsilon}_d)(\vec{S}' \cdot \hat{p}) + c_3^t(\vec{\epsilon}_\gamma \cdot \vec{S}')(\vec{\epsilon}_d \cdot \hat{p}), \\ \mathcal{M}_{K^-d \rightarrow K_1K_2B_1B_2} &= d_1^s(\vec{\epsilon}_d \cdot \hat{p}) + d_2^t(\vec{\epsilon}_d \times \vec{S}') \cdot \hat{p}. \end{aligned} \quad (\text{A5})$$

Here a , b , c , and d are some functions of s and of m (and, in general, of further invariants that are required to specify the kinematics of the reaction), where their upper indices indicate whether they correspond to spin-singlet (s) or spin-triplet (t) amplitude. The polarization vectors of the deuteron and photon are denoted by $\vec{\epsilon}_d$ and $\vec{\epsilon}_\gamma$, respectively. The spin vectors \vec{S} , \vec{S}' are used for the spin-triplet initial and final states, respectively. For the last two reactions we assume the momenta of the final kaons to be either aligned or anti-aligned with the direction of the initial CM momentum \hat{p} . This leads to a significant simplification allowing one to separate different spin states. For the reaction $K^-d \rightarrow KB_1B_2$ such a restriction is not necessary and the momentum of the emitted kaon is denoted by \vec{k} . For the reaction $pp \rightarrow K_1K_2B_1B_2$ we assume for simplicity that both emitted kaons go into the same direction \hat{k} . It is convenient to introduce the following set of polarization observables

$$\begin{aligned} O_1 &= (1 - \sqrt{2}T_{20}^0) \frac{d\sigma_0}{dm^2dt}, \quad O_2 = (2 + \sqrt{2}T_{20}^0) \frac{d\sigma_0}{dm^2dt}, \quad O_3 = T_{10}^0 \frac{d\sigma_0}{dm^2dt}, \\ O_4 &= \left(2 + \sqrt{2}T_{20}^0 + \sqrt{3}(T_{22}^l + T_{2-2}^l)\right) \frac{d\sigma_0}{dm^2dt} = \sqrt{3}(-\sqrt{2}T_{10}^c + (T_{22}^l + T_{2-2}^l)) \frac{d\sigma_0}{dm^2dt}, \\ O_5 &= A_{0y} \frac{d\sigma_0}{dm^2dt}, \quad O_6 = (1 + A_{yy}) \frac{d\sigma_0}{dm^2dt}, \end{aligned} \quad (\text{A6})$$

where the various T 's for the γd initial state are defined in [30], and $\frac{d\sigma_0}{dm^2dt}$ is the unpolarized differential cross section. T_{20}^0 and T_{10}^0 have the same definition also for the K^-d initial state, the only difference being the absence of the summation over the photon polarizations. The observable O_1 selects the amplitudes with longitudinal target polarization $\vec{\epsilon}_d \parallel \hat{p}$, whereas O_2 , O_3 , O_4 select the amplitudes with $\vec{\epsilon}_d \perp \hat{p}$. In addition O_4 contain only that part of the amplitude which is antisymmetric with respect to an interchange of $\vec{\epsilon}_d$ and $\vec{\epsilon}_\gamma$. The observables O_5 and O_6 correspond to the proton-proton induced reaction. Here A_{0y} is the analyzing power and A_{yy} is the spin correlation coefficient for polarized beam and target [28, 43], and y is the direction perpendicular to the reaction plane.

Now inspecting the structure of the reaction amplitudes (A5) we can identify the observables that allows one to separate a particular spin state: For the reaction $K^-d \rightarrow KB_1B_2$ the triplet final state can be singled out by the observable O_1 for any direction of the emitted kaon, or one can measure the unpolarized differential cross section for $\vec{k} \parallel \hat{p}$. The spin singlet state cannot be separated. For the reaction $\gamma d \rightarrow K_1K_2B_1B_2$ the triplet final state can be separated by measuring O_1 . The observable O_4 provides access to the spin-singlet amplitude. For the reaction $K^-d \rightarrow K_1K_2B_1B_2$ the observable O_1 separates spin-singlet contribution, whereas O_2 and O_3 separate spin-triplet state. For the reaction $pp \rightarrow K_1K_2B_1B_2$ the observable O_6 separates the spin-triplet contribution, whereas O_5 is proportional to $\sin\theta \Im\{b_1^s b_2^{s*} + b_4^t b_5^{t*} \cos\theta\}$ with $\cos\theta = \hat{p} \cdot \hat{k}$. Since the b_i 's are even functions of $\cos\theta$ (due to parity conservation), after the integration of O_5 over an angular region symmetric with respect to $\theta = \frac{\pi}{2}$ only spin-singlet amplitudes survive.

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